

How to Measure the Earth

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Introduction

Who first determined the size of the Earth? How did they do it? These fundamental questions arise in studying early Greek, Indian and Islamic mathematical astronomy. In this article we look at the attempts of Eratosthenes, Posidonius, and al-Bīrūnī to determine the circumference of the Earth and ways to use this topic in the classroom. These calculations use only basic knowledge of geometry and trigonometry, so that instructors in many different courses can include this topic in their syllabus. It would be appropriate to discuss the problem in a high school or college geometry class, in a precalculus class, a history of mathematics class, or in a freshman mathematics survey class.

There are three primary methods for determining the circumference of the Earth: using the lengths of shadows, the elevation of stars, or the altitude of a mountain. Explaining these methods can be done in roughly two hours of class time. If an instructor wants to assign students a project to carry out one of these calculations then one or two more hours may be needed to complete the topic (assuming that the students do measurements during class time).

There are certain geographical and astronomical terms that are frequently used in this topic and should be defined for students. The position of a point on the Earth's surface is given by two coordinates, its latitude and longitude. The *latitude* of a point is measured by how far it is north or south of the equator, so that points of equal latitude form a circle parallel to the equator. Latitude is measured in degrees from the equator (0°) to the poles (90°). *Longitude* measures how far east or west the point lies. Points on a semicircle passing between the poles form what is called a meridian. Today we measure longitude relative to the meridian passing through Greenwich, England called the prime meridian. Longitude is measured in degrees from 0° at the prime meridian to 180° east or west.

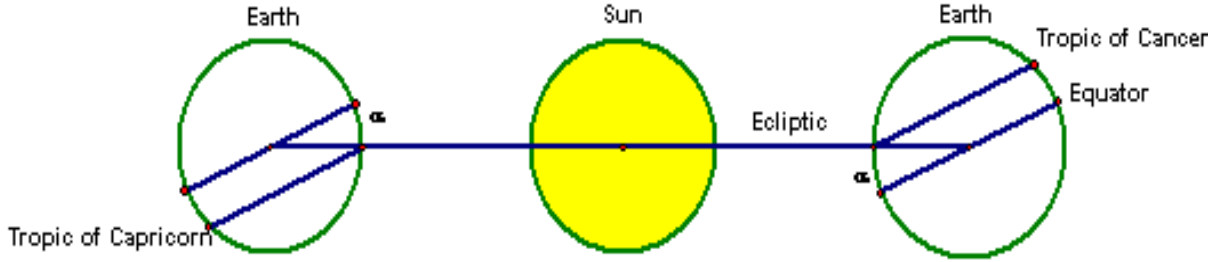


Figure 1: The obliquity of the ecliptic

The apparent path of the Sun through the sky is called the *ecliptic*. The plane in which the Earth orbits around the Sun is called the ecliptic plane, although the early astronomers that we will discuss generally believed that the Sun orbited the Earth. The Earth's axis of rotation is tilted with respect to the ecliptic, see Figure 1. In that figure, the angle α represents the angle between the Earth's equator and the plane of the ecliptic. This tilt is called the *obliquity* of the ecliptic and measures approximately $23^\circ 26'$. Because of the Earth's tilt, the place in the northern hemisphere where the Sun is directly overhead at noon on the summer solstice, is not on the equator but at a latitude called the Tropic of Cancer (the winter solstice and the Tropic of Capricorn play an equivalent role for the southern hemisphere). This implies that the obliquity of the ecliptic is equal to the latitude of the Tropic of Cancer.

Eratosthenes determined that the arc between the Tropic of Cancer and the equator was $11/83$ of a meridian. This translates into a tilt of $23^\circ 51' 20''$, a very accurate result. Further discussion of the obliquity calculation can be found in [10, 11]. A general reference for the history of ancient astronomy is the treatise of Evans [6].

Historical Introduction

In this section we consider the problem of how we know that the Earth is a sphere and then present the methods of Eratosthenes, Posidonius, and al-Bīrūnī to measure the circumference of the Earth. Note that the methods of Eratosthenes and Posidonius compute the polar circumference (i.e. along a longitude) while the method of al-Bīrūnī measures the circumference of some great circle (although this method can usually be arranged to compute a polar circumference).

Why is the Earth a sphere?

Before one can determine the circumference of the Earth there must be a prior assumption that the Earth is a sphere. Asking students to give reasons why the Earth is spherical makes for an interesting class discussion (the instructor should be ready for surprising responses).

Greek, Indian and Islamic astronomy accepted the hypothesis of a spherical Earth, while Chinese astronomy ascribed to a flat-world theory. How did this idea of a spherical Earth arise? Perhaps from the Pythagoreans who believed that the sphere was the most perfect shape, hence all celestial bodies were spheres. Aristotle gave three empirical arguments why the Earth is round.

- Matter is drawn to the center of the Earth.
- As you move north to south, new constellations are seen in the sky.
- During a lunar eclipse, the Earth's shadow on the Moon is round.

Aristotle stated in *De Caelo* Book II, "Also, those mathematicians who try to calculate the size of the Earth's circumference arrive at the figure 400,000 stades ¹ This indicates not only that the Earth's mass is spherical in shape, but also that as compared with the stars it is not of great size." This clearly indicates that the Greeks had a basic understanding of geographical and astronomical distances. They had a rough idea of the size and shape of the Earth, of the distances between Earth, Moon, and Sun, and the fact that the stars are far away. Another interesting topic for classroom discussion is the problem of how we know the stars are very far away (compared to distances within the Solar System).

Of course the Earth is not a perfect sphere, but instead is an oblate spheroid, being slightly flattened at the poles and bulging at the equator. The circumference of the equator (24,902.4 miles) is greater than the polar circumference (24,860.2 miles). This fact was reported by Pierre de Maupertuis in 1737 [13].

As we will discuss in the next section, the circumference of 400,000 stades reported by Aristotle is far too large. A more accurate computation was done in the century after Aristotle by Eratosthenes.

¹The stade is the standard Greek unit of distance, representing the length of a foot race. As discussed below, the actual length of a stade is highly disputed.

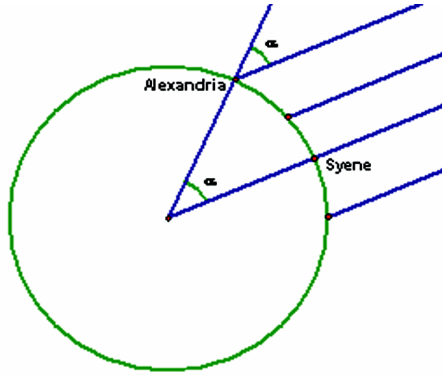


Figure 2: The method of Eratosthenes

How to measure the Earth using shadows

The earliest method for computing the circumference of the Earth for which we have a detailed account is that of Eratosthenes (276 BCE - 194 BCE). Born in the northern African city of Cyrene, Eratosthenes was known for his accomplishments in a wide variety of disciplines. He was a noted astronomer, mathematician, geographer, librarian, poet, and philosopher. Eratosthenes followed his teacher Callimachus in becoming the librarian at the famed library of Alexandria.

The calculation of the Earth's circumference appeared in the now lost work, *On the measurement of the Earth*. Later descriptions of this work appear in commentaries by Cleomedes, Strabo and Pliny. The method of Eratosthenes consists of the following steps.

1. Choose two locations a known distance apart, with the same longitude, but different latitudes. Eratosthenes used the Egyptian cities of Alexandria and Syene (the present-day Aswan).
2. Next, simultaneously measure the angle of inclination of the Sun at both locations. Here it is assumed that the Sun is so distant that all rays of light hitting the Earth are parallel. This implies that the difference in the angle of the Sun at these two locations will equal the difference in their latitude. Eratosthenes believed that Syene was on the Tropic of Cancer, so that at noon on the summer solstice the Sun is directly overhead at Syene. Hence the angle of the Sun measured at noon in Alexandria on the summer solstice will equal the difference in latitude of the two cities, see Figure 2.
3. Then use the following formula to determine the circumference of the Earth,

$$(1) \quad \frac{\text{circumference}}{\text{distance between locations}} = \frac{\text{angular measure of a circle}}{\text{angular difference of latitudes}}.$$

Thus, in order to compute the circumference of the Earth, Eratosthenes only needed to know two data values: the angle of the Sun in Alexandria at noon on the summer solstice and the distance between Alexandria and Syene.

How did Eratosthenes measure the angle of the Sun? Presumably Eratosthenes used a gnomon (a vertical rod) to measure the shadow's length. In Figure 3, h is the height of the gnomon, s the length of the shadow. The angle of the shadow α can be computed by the formula

$$\alpha = \arctan \frac{s}{h}.$$

Of all the current methods to measure the inclination of the Sun, the gnomon is the simplest to use in the classroom. Students can easily construct a gnomon, measure the length of the shadow at noon and compute the above arctangent. But how did Eratosthenes compute the inverse tangent? The ancient Greeks did not have trigonometry as we know it today. Hipparchus in the next century developed a table of chords, but not a triangle based theory. It seems more likely that Eratosthenes used a type of graduated sundial known as a *skaphe*. This is a hemisphere with a gnomon in its center and markings on the inside of the bowl which could be used to determine the angle of the Sun.

Eratosthenes found that the noon-day shadow at Alexandria traversed $1/50$ of a circle, which translates to an angle of $7^\circ 12'$. There are several sources of error in this calculation. In reality Syene is $21'$ north of the Tropic of Cancer. Also, Syene is 3° east of Alexandria. The correct difference in latitudes is $7^\circ 7'$. Despite these errors, the value given by Eratosthenes, $7^\circ 12'$, is only slightly more than 1% in excess of the true value.

The other piece of data that Eratosthenes needed for the circumference calculation is the distance between Syene and Alexandria. He used a value of 5000 stades for this distance. It's not clear how this distance was determined. Perhaps it was an accepted figure based on existing data of traders sailing on the Nile between the cities. Another possibility is that the distance was calculated by *bematists*, specialists trained to measure distance by counting their paces as they walked.

Assuming the figure of 5000 stades and the ratio of $1/50$ for the arc of the Earth's surface between Alexandria and Syene, Eratosthenes would have computed the Earth's circumference to be $5000 \cdot 50 = 250,000$ stades. Cleomedes gives this result [14, vol. 2, pp. 267-273], while Strabo and Pliny give 252,000 stades as the figure computed by Eratosthenes. It has been speculated that the figure of 252,000 was used because it is divisible by 360,

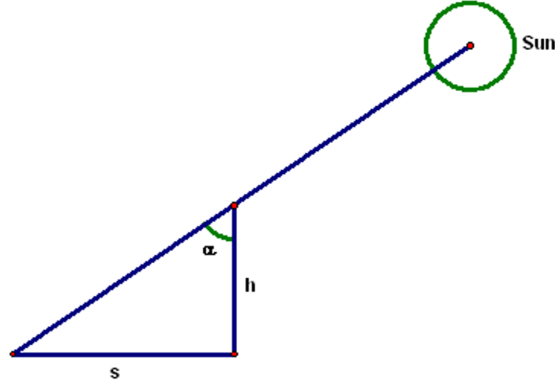


Figure 3: Angle of Sun using a gnomon

thus giving a figure of 700 stades per degree of meridian.

How long is a stade? The stade is a unit of distance related to the length of a sprint in ancient Greek athletic competitions. In different locations, different distances were used for the race. One scholar determines that the stade used by Eratosthenes is 166.7 m [8], while another, citing the testimony of Pliny that 40 stades equaled 5 Roman miles, gives a stade as 184.98 m [5].

If we use the larger figure of 184.98 m. then Eratosthenes' 250,000 stades translates to 46,245 km (28,735.31 miles), which is nearly 16% in excess compared to the true polar circumference of 40,008.6 km (24,860.2 miles). Whereas, if we use the figure of 166.7 m for a stade then 250,000 stades equals 41,675 km (25,895.64 miles), which is only 4% in excess of the correct value. We may never know the precise value of Eratosthenes' stade, but in any case it is clear that the accuracy of his calculation of the Earth's circumference is extremely impressive.

How to measure the Earth using stars

The Stoic philosopher and mathematician, Posidonius of Rhodes (135 BCE - 51 BCE), used a method that is slightly different from that of Eratosthenes [4]. Consider the cities of Rhodes and Alexandria, which are on the same meridian (more or less). In Rhodes, the star Canopus just appears on the horizon (actually it reaches a height of 1° above the horizon). In Figure 4 this is shown by Canopus being on the tangent to Rhodes. As one sails south from Rhodes, Canopus appears higher and higher in the sky. At Alexandria, the star appears at a maximum altitude of $7^\circ 30'$ above the horizon.

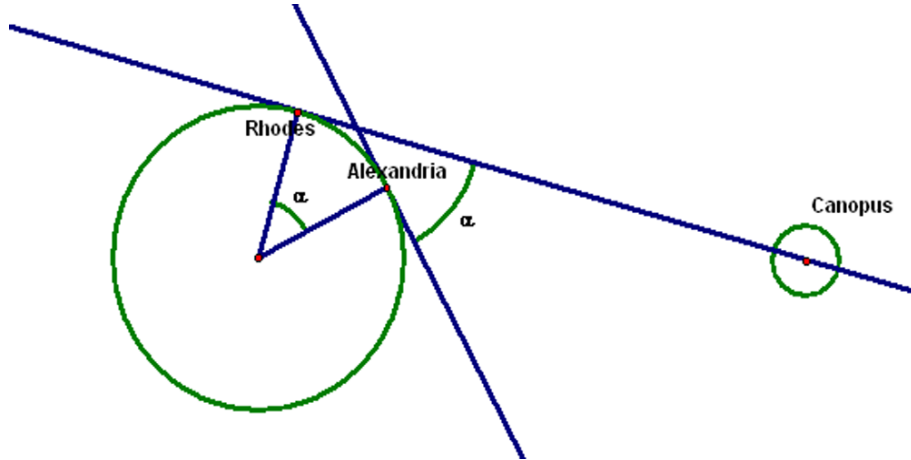


Figure 4: The method of Posidonius

Posidonius takes the angle of elevation of Canopus at Alexandria to be α in Figure 4. This angle also corresponds to the difference in latitudes of the cities. This may seem puzzling, for it would appear by looking at the figure that the angle of elevation of Canopus at Alexandria is in fact larger than α . But Posidonius is assuming that the distance from the Earth to Canopus is far greater than the distance between Rhodes and Alexandria, which is of course true. In this case, the angle of elevation at Alexandria will be practically equal to α .

Posidonius used 5000 stades as the distance between Rhodes and Alexandria (leading to the hypothesis that all Greek cities at the time were 5000 stades apart). This gives a computed circumference of

$$5000 \cdot \frac{360}{7.5} = 240,000 \text{ stades.}$$

Strabo instead stated that Posidonius found the circumference to be 180,000 stades, making it the smallest such measure from antiquity. Some scholars believe that this shorter distance was used by Columbus as an argument to sponsor his journeys, i.e India is not really all that far away.

The discrepancy seems to result from an error in Posidonius' calculation. The distance of 5000 stades between Rhodes and Alexandria is too large. Strabo says that Eratosthenes determined the distance to actually be 3750 stades. Using this distance instead of 5000 stades leads to the circumference of $3750 \cdot 360/7.5 = 180,000$ stades, as given by Strabo.

If we accept the value of 240,000 stades, this converts to 40,008 km (24,859.82 miles), which is only 0.001% smaller than the current value of 40,076.5 km. Given the errors in Posidonius' calculations, this is a rather amazing coincidence! The article of Fischer [7] has a good discussion of the methods of Eratosthenes and Posidonius.

How to measure the Earth using mountains

The Islamic mathematician Abu Arrayhan Muhammad ibn Ahmad al-Bīrūnī (973 - 1048) developed a method for calculating the circumference of the Earth by sighting the horizon from the top of a mountain of known height. He was a noted Islamic scholar, mathematician, astronomer, physician and astrologer. Al-Bīrūnī was born in the region of Khwarazm, in present day Uzbekistan. He studied and later collaborated with the mathematician and astronomer Abu Nasr Mansur. He and Mansur carried on scientific studies under the patronage of the brothers Abu'l Abbas and Ali ibn Ma'mun. The Ma'muns were later overthrown by Sultan Mahmud who then became al-Bīrūnī's patron, or perhaps captor; the relationship isn't clear.

Mahmud led a military campaign to India and took al-Bīrūnī with him. While in India, al-Bīrūnī studied the language, literature, customs, religion and scientific achievements of that country. As a result of his studies he wrote the treatise *India - Containing an Explanation of the Doctrines of the Indians* [3].

Our discussion of al-Bīrūnī's calculation of the circumference of the Earth is drawn from his treatise *The Determination of the Coordinates of Positions for the Correction of Distances between Cities* ("Kitāb Taḥdīd Nihāyāt al-Amākin Litaṣṣih Masāfāt al-Masākin") [2]. Another source for this material is Berggren [1, pp. 141-143].

Al-Bīrūnī presents three methods for determining the circumference of the Earth, [2, pp. 183-189]. We will study the second method, which uses a mountain of given height. Before introducing his method, al-Bīrūnī states, seemingly as a comment on the method of Eratosthenes,

Here is another method for the determination of the circumference of the Earth. It does not require walking in deserts.
[2, p. 183]

Assume the existence of a mountain of known height, represented by the segment EL in Figure 5. As you stand at the top of the mountain, at point E , you sight the horizon along line ET using an instrument, such as a theodolite, that can measure angles. In its simplest form a theodolite can be made by attaching a protractor to a sighting tube. In the figure, circle ABZ may be thought of as representing the protractor, held vertically. The actual radius of the protractor, admittedly rather large in the figure, is irrelevant to the calculation. The angle $\alpha = \angle BET$, called the dip angle, will be found by this observation. Draw line MZ perpendicular to EL . We wish to find KT , the radius of the Earth.

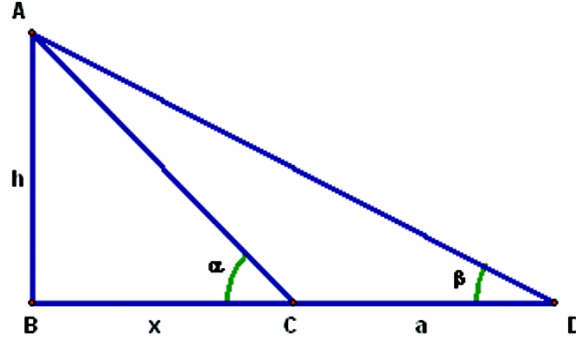


Figure 6: Computing the height of a mountain

Then the Earth's radius KT can be computed from Equation (2). To summarize, the radius of the Earth, KT , is given by

$$(6) \quad KT = EK \times \frac{ZM}{EZ} = EL \times \frac{\cos \alpha}{1 - \cos \alpha}.$$

The presence of trigonometric functions in this formula did not cause al-Bīrūnī any problem at all. He made highly accurate tables of the sine, cosine, and tangent functions.

This method has an advantage over that of Eratosthenes in that it doesn't require measurements at two different locations. But on the other hand, in practice $\angle EZM$ will be extremely small unless the mountain is very high. For example, for a mountain of height 1000 ft, $\angle EZM = 0^\circ 34'$. Even for a mountain of 10,000 ft the dip angle is still quite small, $\angle EZM = 1^\circ 46'$. And if al-Bīrūnī were an expert mountain climber and measured the angle from some Himalayan peak 20,000 ft high, the angle would only be $2^\circ 30'$.

The above calculation requires that we know the height of a mountain. How did al-Bīrūnī calculate this height? He gives more than one method for computing the height of mountains. We consider a method which involves a very simple calculation but requires two sightings of the mountain, see [12]. For another procedure that only requires one sighting, but a somewhat more difficult calculation, see [2, pp. 187-188] or [1, pp. 141-142]. In Figure 6 the height of the mountain which we wish to determine is $h = AB$. Make one sighting of the mountain top from point C , so that the angle $\alpha = \angle ACB$ is determined. Make another sighting from point D , computing the angle $\beta = \angle ADB$. It is necessary to assume that the two locations C, D are themselves at the same height. Al-Bīrūnī computes h by

$$(7) \quad h = \frac{a}{\cot \beta - \cot \alpha}.$$

This follows since

$$\cot \beta - \cot \alpha = \frac{x + a}{h} - \frac{x}{h} = \frac{a}{h}.$$

While this formula is very simple, if the points C and D are not far apart then the angles α, β will be practically indistinguishable and the calculation of h may involve considerable round-off error.

Al-Bīrūnī actually used his method to compute the Earth’s circumference. On one of his journeys to India with Sultan Mahmud, al-Bīrūnī stayed at Nandana Fort, which is in the Punjab region of present-day Pakistan. According to some accounts al-Bīrūnī was being held in detention. He found a nearby peak that was convenient for his calculations. It had an open view on the plains to the south. The peak is 1795 ft above sea level, but al-Bīrūnī measured the height from the surrounding plains. He found the height above the plains to be 1055.18 ft. The unit of measurement that al-Bīrūnī used was the cubit. The height is equal to 652.055 cubits (using the conversion of 1 cubit = 1.61825 ft found in [12]). Al-Bīrūnī measured the dip angle to be $0^\circ 34'$. Using Equation (6), al-Bīrūnī finds a radius of 12803337;2,9 cubits, al-Bīrūnī uses sexagesimal notation for the fractional part. The radius translates into 3924.05 miles, which is only 0.6% smaller than the true value. This leads to a circumference of 25,044.99 miles, which is 0.7% larger than the true value of 24,860.2 miles. This calculation seems suspiciously accurate, perhaps due to the conversion from cubit to feet.

In the Classroom

There are several projects that may be assigned on this topic. Each project requires students to not only understand the underlying mathematics but also deal with the problems of applying mathematics in the real world.

Sample Projects

- Measure the circumference of the Earth using Eratosthenes’ method. In order to use the method of Eratosthenes in the classroom there are several practical issues to consider.

It is recommended that the students use a gnomon to measure the length of shadows. Gnomons are simple enough for almost any student to construct. A potential problem in using a gnomon is to ensure that it is vertical. This can be accomplished by using a level while constructing and using the gnomon.

Since my school isn’t in session on the summer solstice (or winter solstice for those in the Southern Hemisphere), when do I take measurements? There are a couple of solutions to this problem.

- Using one measurement of the Sun: The easy way to do this is to first look up the distance between your location and the Tropic of Cancer. This then requires only one measurement of the Sun's inclination. Instead of using noon at the summer solstice one can use noon on the vernal or autumnal equinoxes. On either of these dates, the Sun is directly overhead at noon on the Equator. So if the students measure the Sun's inclination at noon on an equinox (conveniently each semester has an equinox) and if the distance between your present location and the Equator is known then Equation (1) can be used to compute the circumference. Of course having to look up the distance to the Equator may seem like cheating. So the next method is more in the spirit of Eratosthenes.
- Using two measurements of the Sun: This method is a lot more flexible in terms of scheduling the project (this method can be used at any time of year during daylight), but one needs to measure the Sun's inclination from two locations. The locations have to be on the same longitude, have to be a known distance apart, and must be far enough apart for the difference in angle of inclination to be detectable. All that needs to be done is to schedule two observations of the Sun at the same time on the same day in different locations on the same meridian. In order for this method to be practical one either needs to be conveniently located near a road that runs due north or south, or one must find a companion school on the same latitude to collaborate with.

If measurements are supposed to be taken when the Sun is the highest in the sky (namely, solar noon) then when exactly should the observations be done? The simplest way to compute solar noon is to take the midpoint of the local times for sunrise and sunset (which can be found in any newspaper).

- Measure the circumference using the method of Posidonius. How would one use this method in the classroom? Here are some of the issues that need to be addressed.
 - One obvious problem is that it involves night observations. There is no way to get around this problem.
 - How should one measure the elevation of stars in the night sky? The simplest way to measure this angle is to use a sextant. These

are reasonably easy to construct; for further discussion see the Web site:

<http://www.tecepe.com.br/nav/XTantProject.htm>.

- A major issue is that the instructor is probably not as fortunate as Posidonius. In order to do the calculation making only one measurement, one needs to find a star that rises only up to the horizon in some city that is on the same longitude and a known distance from one's current location.
 - The previous point means that the calculation will likely require two simultaneous measurements from two locations on the same meridian and a known distance apart. How far apart should the measurements be? A distance of 300 miles corresponds to a difference in the angle measured of approximately $3^{\circ}46'$. Using a distance much smaller than this requires highly accurate measurements.
- Measure the circumference using a variation of Posidonius' method. The earliest known attempt to measure the Earth was by Archytas, who was a contemporary of Plato. The method used by Archytas was to find two cities on the same meridian, each city having a star directly over their locations at the same time of night. The angular distance between the two cities is then equal to the angle between the stars along a celestial meridian.
 - Measure height of an object using the method of al-Bīrūnī discussed above. For example, measure the height of a hill, mountain, tall building, tethered balloon or other tall object. A word of caution is necessary. It is preferable to use an object that has a well-defined peak (a hill can have a too rounded top to be useful). The two points from which to make your sightings have to be fairly far apart (the farther the better). If possible, take the first sighting close to the base of the object. In my history of mathematics class, I used the four story classroom building in which the class is taught.
 - Measure the circumference of the Earth using the method of al-Bīrūnī. This presents some difficulties when used in the classroom.
 - The main difficulty may be the need for a height that not only can be measured but that can also be climbed in order to measure the dip angle. If the instructor is hindered by the lack of a convenient

mountain nearby the classroom, a sufficiently tall building may be substituted for the mountain.

- Even accounting for the availability of a nearby height, another obstacle is how to accurately measure the very small dip angle. One strategy to alleviate this problem is to make several sightings and compute the average dip angle.
 - In any case, one needs an accurate instrument for measuring angles. Either a sextant or theodolite may be constructed for this purpose. It is to be expected that a homemade instrument will be rather inaccurate and impact the final value for the circumference.
- Determine how to compute height using only one sighting. This is described in detail in Berggren [1, pp. 141-142].
 - Make your own surveying instrument (such as sextant, theodolite, astrolabe, or sundial) and then use this instrument in making real world measurements. There is a variety of interesting mathematics involved in constructing these instruments. Here are some Web resources for making such instruments.

Making your own sextant:

<http://www.tecepe.com.br/nav/XTantProject.htm>

Making your own theodolite:

<http://www.dar.csiro.au/airwatch/awballoon.html>

<http://www.apogeerockets.com/education/downloads/newsletter93.pdf>

Making your own astrolabe:

<http://www.astrolabes.org/>

<http://celebrating200years.noaa.gov/edufun/book/MakeyourownAstrolabe.pdf>

Making your own sundial:

<http://www.lmsal.com/YPOP/Classroom/Lessons/Sundials/sundials.html>

Making your own vertical sundial:

http://www.mysundial.ca/sdu/graphical_vertical_sundial.html

- Determine the latitude or longitude of your location. One way to compute latitudes in the northern hemisphere is to use the elevation of the North Star. This star is more or less directly overhead at the North Pole, so that at any location in the northern hemisphere the angle of the North Star's elevation will equal that point's latitude.

Computing longitude is more difficult. The relative longitude of two locations can be determined by comparing the time at which a lunar eclipse occurs at both locations. The ratio of the difference in longitude to 360° equals the ratio of the difference in time of the eclipse to the length of a sidereal day, which is the time it takes for stars to return to their highest point (a little less than 24 hours).

- For high school students an interesting organized activity is the online Eratosthenes Experiment,

<http://www.youth.net/eratosthenes/>.

This is a world-wide project in which students from around the world measure the angle of the noon-day Sun. Using this information together with the school's latitude and longitude allows the students to calculate the Earth's circumference. The experiment takes place twice a year, on the spring and autumn equinoxes.

Taking it Further

In this article we have examined various methods for measuring terrestrial distances. This discussion may be extended to the measurement of distances between the Earth and other heavenly bodies. This uses the concept of parallax, which is the apparent angular displacement of an object, due to the motion of the observer. There are three common uses of parallax: to measure the distance between the Earth and the Moon, between the Earth and the Sun, and between the Earth and nearby stars. The use of parallax in measuring astronomical distances can be found in the work of Greek mathematicians such as Aristarchus, Hipparchus and Ptolemy. A good history of the use of parallax in astronomy can be found in the survey of Hirshfeld [9].

Both the solar and stellar parallax are too small to measure without refined instruments, but it is feasible to measure the lunar parallax as a class project and hence to determine the distance to the Moon. Lunar parallax involves the different position of the Moon relative to the background of fixed stars when the Moon is viewed simultaneously from two different locations on Earth. In fact, the method of comparing sightings of the Moon from two different locations on Earth is very reminiscent of the procedure of Eratosthenes.

Conclusion

Let us conclude with an object lesson. Given the lack of technology available to these early astronomers, it is remarkable how truly accurate their calculations were. But such accomplishments also come with a classical admonition against hubris. Archytas, famed for his achievements in mathematics, astronomy and philosophy was the subject of a renowned ode of Horace (Odes I.28). The beginning of the poem states the theme that all of the triumphs of Archytas are made trivial by the great leveler, death.

Archytas, you who measured the Earth and the sea and the
numberless sands,
are now confined in a small mound of dirt near the Matine shore,
what does it avail you that you once
explored the mansions of the skies and that you traversed
the round celestial vault – you with a soul born to die?

References

- [1] Berggren, J. L., *Episodes in the Mathematics of Medieval Islam*, Springer Verlag, New York, 1986.
- [2] al-Bīrūnī, *The Determination of the Coordinates of Positions for the Correction of Distances between Cities*, tr. Jamil Ali, Centennial Publications, American University of Beirut, 1967.
- [3] al-Bīrūnī, *India - Containing an Explanation of the Doctrines of the Indians*, tr. as *alBeruni's India* by E. Sachau, Kegan Paul, Trench, Trübner & Co., London, 1910.
- [4] Drabkin, I. E., “Posidonius and the Circumference of the Earth”, *Isis*, **34**(6) (1943), 509-512.
- [5] Engels, D., “The Length of Eratosthenes’ Stade”, *Amer. J. Philology*, **106**(3) (1985), 298-311.
- [6] Evans, J., *The Theory and Practice of Ancient Astronomy*, Oxford University Press, Oxford, 1998.
- [7] Fischer, I., “Another Look at Eratosthenes’ and Posidonius’ Determinations of the Earth’s Circumference”, *Quart. Jnl. Roy Astro. Soc.*, **16**(1975), 152-167.

- [8] Gulbekian, E., “The Origin and Value of the Stadion Unit used by Eratosthenes in the Third Century B.C.”, *Arch. Hist. Exact. Sci.*, **37**(4) (1987), 359-363.
- [9] Hirshfeld, A. W., *Parallax: The Race to Measure the Cosmos*, W.H. Freeman, New York, 2001.
- [10] Jones, A., “Eratosthenes, Hipparchus, and the Obliquity of the Ecliptic”, *J. Hist. Astron.*, **33**(2002), 15-19.
- [11] Rawlins, D., “Eratosthenes’ Geodesy Unraveled: Was There a High-Accuracy Hellenistic Astronomy?”, *Isis*, **73**(2) (1982), 259-264.
- [12] Rizvi, S., “A Newly Discovered Book of Al-Bīrūnī: “Ghurrah-uz-Zijāt”, and al-Bīrūnī’s Measurements of Earth’s Dimensions”, *Al-Bīrūnī Commemorative Volume*, ed. H. Said, Hamdard National Foundation, Karachi, Pakistan, 1979, 605-680.
- [13] Terrall, M., *The man who flattened the Earth : Maupertuis and the sciences in the enlightenment*, University of Chicago Press, Chicago, 2002.
- [14] Thomas, I., *Selections Illustrating the History of Greek Mathematics*, Loeb Classical Library, Harvard University Press, 1941.